# D1.1.T-matrix of spheres and pillars

Date: 14 October 2022

D.1.1: T-matrix of spheres and pillars
WP1. T.1.1: Modelling of phononic surfaces
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# 1. Technical References

## 1.1. Project General Information

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## 1.2. Version History

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# 2. Table of Contents

# 3. Executive Summary

In this document we provide the expressions for the scattering properties three types of scatters attached to a thin elastic plate: head-neck pillars, single pillars, and spheres. These expressions are useful to analyse the multiple scattering of flexural waves on thin elastic plates, and we will use them for the analysis of Rayleigh waves and other SAWs.

# 4. Introduction

The samples to be fabricated and characterized during the execution of Dynamo are especially complex to study theoretically. The reason of that is that they will contain many scatterers for surface acoustic waves (SAWs) arranged in quasi-periodic or random patterns. Furthermore, the scatterers – under certain conditions (e.g., at high frequencies) – may exhibit mode-direction-dependent scattering properties. Consequently, the widely used theory of periodic structures (with Bloch theorem as the basis) is not suitable in the study of these samples, as are some of the techniques based on the point impedance models. Hence, a different approach must be employed.

The first proposal in Dynamo, given the expertise of the theoreticians of the consortium, was to develop a multiple scattering theory version for these waves. This theory has been successfully applied for the study of complex arrangements of scatterers for flexural waves, and we considered that its extension to more complicated types of SAWs might be straightforward.

# 5. T Matrix of scatterers attached to an elastic surface

The first step towards the development of this theory is the derivation of an analytical expression for the T-matrix of a pillar deposited over a thin elastic plate. We already know the scattering properties of point-like resonators, characterized by a couple of parameters (its mass and its spring constant, and – if required – by the damping parameter), so that the most obvious approach is to compute the equivalent mass and spring constant of a pillar and use that simplified discrete model for studying continuous systems (like pillars) within a selected frequency band. Interestingly, in the literature we can find lumped models for thin pillars (neck) with a larger cylindrical head on the top[i]

$$m_0 = \frac{\pi \rho_h d_h^2 h_h}{4},$$
$$k_0 = \frac{E_n \pi d_n^2}{4 h_n}.$$

or for spheres[ii], where the mass is taken as the mass of the sphere and the spring constant is experimentally obtained from the resonant frequency of the sphere.

Unfortunately, the neck-head model fails when pillars alone are considered, and it is obvious that the neck-head structure is much more complex to fabricate at the nanoscale, so that an alternative model might be found.

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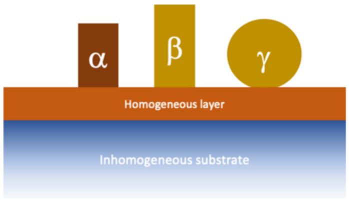
In a forthcoming publication will be found the derivation of the equations for the effective mass and spring constant of a pillar attached to a thin elastic plate, which are given below:

$$m_0 = \frac{8\rho a^2 L}{\pi},$$
$$k_0 = \frac{2E\pi a^2}{L}.$$



We have successfully applied this model in the design of special clusters of scatterers capable of sustaining the so-called Bound States in the continuum (BICs), and we are working now on an experimental demonstration of these states.

In the above result we have considered that the pillars only oscillate longitudinally (compressional mode) however our experience shows that pillars also oscillate transversely (flexural mode). This complex interaction is the result of the coupled out-of-plane and in-plane displacements and rotations, which – depending on the assumptions on the substrate model (e.g., Euler-Bernoulli/Timoshenko/Lamb or other models) and the assumptions on the scatterers (e.g., translational/rotational impedances) – requires detailed analysis. Although we have worked on an extension theory to include this movement, the excitation of other plate modes is unavoidable in this situation, and we believe that the theory will always be incomplete.

Therefore, we concluded that, although the T-matrix will give us excellent results for compressional modes, we could explore other possibilities for more complicated structures. The two possibilities that we will consider are briefly explained below. The first one has given excellent results for photonics and acoustics, while the second one has given excellent results for photonics.

## 6. Mode matching approach

The mode matching method is a semi-analytical theory which allows to simplify considerably the search of eigenmode of complex structures whose solutions we know individually. For instance, in the figure below we have a cluster of scatterers attached to a homogeneous layer which has a generally inhomogeneous substrate at the bottom:

We can assume that we know the solution for the elastic field in each domain of the figure. The mode matching method works on the boundaries connecting the different domains to "match" boundary conditions and find a general solution for the joint domain. The great advantage of this method is that if in the scatterers there is only one excitation (as is usually the case) we can reduce the eigenvalue equation to a system of NxN equations, with N being the number of scatterers. This reduces dramatically the numerical complexity of the problem, and convergence can be even faster than with the multiple scattering theory.

Thus, this will be the strategy that we will follow within the next months for deliverable D1.2 (deadline February 2024).

# 7. Guided mode expansion method

When the scatterers patterning the surfaces are holes instead of deposited pillars or spheres, neither the T-matrix approach nor the mode-matching method are suitable for their study. The former because a multipolar theory is required, the latter because there is no field inside the holes and propagation of waves occurs only through the substrate. In this specific case, we found in the literature an alternative method which has been successfully employed in photonics but still needs some optimization for being properly used in elastodynamics.

The fundamental idea consists in expanding the solution of the elastic field as a linear combination of eigenmodes of the substrate without inclusions, and then solve the system either variationally or by the plane wave expansion method (for periodic structures). The convergence is found to be faster given that boundary conditions at the top of the plates are already satisfied.

We will try to generalize this method to other type of substrates, since so far it has been applied only to Lamb waves.

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[i] Chaunsali, R., Chen, C. W., & Yang, J. (2018). Subwavelength and directional control of flexural waves in zone-folding induced topological plates. *Physical Review B*, *97*(5), 054307.

[ii]Boechler, N., Eliason, J. K., Kumar, A., Maznev, A. A., Nelson, K. A., & Fang, N. (2013). Interaction of a contact resonance of microspheres with surface acoustic waves. Physical review letters, 111(3), 036103.